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R. M. Cummings
Associate Editor

Extreme Value Flight Duration Analysis of Four-Engine Spacecraft

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Introduction

As the United States and the other spacefaring nations are preparing a new era of exploration of nearby and distant planets, reliability issues of spacecraft will be studied more closely. New highly reliable craft that will travel long distances are needed. If such craft are crewed, the reliability concern naturally will be greater. This study considers a reliability concern given the fact that each of the engines has a life that is a random variable with known or estimated parameters. Preventive maintenance, such as performed on airplane engines, is not an option on a space voyage. The maiden flight as well as a number of subsequent flights will have to travel nonstop before space stations are built.

If each engine has a random life at some specified operating conditions such as speed and temperature, then the probability of failure and the failure rate will increase over time in the absence of preventive maintenance. Decision makers can install a fixed number (n) of such engines, some or all of which are needed to keep the spacecraft flying. As with airplane counterparts, assume that it may be possible to keep the craft flying even if one or more of the engines have failed. In this Note, n is 4 and there are two extreme types of spacecraft: one that requires all four engines to work (4 out of 4) and one that can fly with only one of the four (1 out of 4) operational. What are reliabilities or probabilities of reaching the destinations for both spacecraft when flight duration is known? For example, it may be desirable to state that the probability of being able to fly at least x time units toward the destination is $\%\ Y$. This probability is the reliability for a fixed flight duration.

Extreme Value Distributions

The topic of extreme value statistic[1–6] is relatively rare in most texts and articles on reliability, but this topic is important in reliability design decisions.[7] It is obvious that system failure is related to its weakest components. If such a component is essential and there are no standby units, then the system will fail as soon as the essential item fails. In a 4-out-of-4 system, for example, the system life distribution is the same as the distribution of the minimum life component. The 1-out-of-4 system, on the other hand, has a life distribution that is the same as the distribution of the maximum life component.

The analytical extreme value distribution can be derived by considering samples of size n, drawn independently and at random from a population described by its probability density function (PDF) or by its cumulative distribution function (CDF). Let the observations drawn be $\{x_1, x_2, \ldots, x_n\}$. Two statistics of interest are

$$y_n = \min\{x_1, x_2, \ldots, x_n\} \qquad \text{and} \qquad z_n = \max\{x_1, x_2, \ldots, x_n\}$$

Distribution of Smallest Values

For the 4-out-of-4 system case,

$$\begin{aligned}\text{Prob}(y_n > y) &= \text{Prob}(\text{all}\, x_i > y)\\ &= \text{Prob}(x_1 > y, x_2 > y, \ldots, x_n > y)\\ &= \text{Prob}(x_1 > y)\text{Prob}(x_2 > y)\cdots\text{Prob}(x_n > y)\\ &= [1 - \text{Prob}(x_1 \le y)][1 - \text{Prob}(x_2 \le y)]\\ &\quad \cdots[1 - \text{Prob}(x_n \le y)]\end{aligned}$$

Therefore, $\text{Prob}(y_n > y) = [1 - F(y)]^n$; the CDF is

$$G_n(y) = \text{Prob}(y_n \le y) = 1 - \text{Prob}(y_n > y) = 1 - [1 - F(y)]^n$$

the reliability function is

$$R_n(y) = 1 - G_n(y) = [1 - F(y)]^n$$

and the PDF is

$$g_n(y) = G_n'(y) = nf(y)[1 - F(y)]^{n-1}$$

Distribution of Largest Values

For the 1-out-of-4 system case, let the CDF be

$$\begin{aligned}H_n(z) &= \text{Prob}(z_n \le z) = \text{Prob}\,(\text{all}\, x_i \le z)\\ &= \text{Prob}(x_1 \le z, x_2 \le z, \ldots, x_n \le z) = [F(z)]^n\end{aligned}$$

The reliability function is

$$R_n(z) = 1 - H_n(z) = 1 - [F(z)]^n$$

and the PDF is

$$h_n(z) = H_n'(z) = nf(z)[F(z)]^{n-1}$$

Simulation

Simulation often helps to bypass complex mathematical work needed to reach the same result. The simulation software[8] used is a powerful tool that helps decision makers handle situations subject to uncertainty. The software[8] performs Monte Carlo simulation. Extensive use of Monte Carlo simulation in reliability work is well documented in the literature. Monte Carlo simulation is a common tool for engineers who, otherwise, face complex mathematics to deal with.

Specific Reliability Structure Under Study

The system consists of four independent engines or components. A mean life of 3000 time units and three distributions are assumed. An exponential distribution is commonly used in reliability work for illustration purposes because this distribution has a constant failure rate irrespective of duration of use. If preventive maintenance is performed, the exponential assumption is often accurate. Two other distributions, normal and uniform, are also used to consider the case when the failure rate increases over time as it would be in a long and nonstop space voyage. However, uniform and normal distributions require additional parameters. The mean life of 3000 is maintained,

Received Oct. 24, 1996; revision received Jan. 2, 1997; accepted for publication Feb. 14, 1997.

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but the other necessary parameters are also determined. For uniform distribution, the upper and lower limits are 2500 and 3500; for normal distribution, the mean is 3000 and standard deviation is 500.

If there are n parallel engines under use until failure, life data would be available as a result of actual use or accelerated life testing conditions.

Let t_i be the random life of the ith unit in the system ($i = 1, 2, 3, \ldots, n$), where

$$\{t_i\} = \{t_1, t_2, t_3, \ldots, t_n\}$$

and let $t_{[j]}$ be the life of the unit that fails in jth sequence ($j = 1, 2, 3, \ldots, n$), where

$$t_{[1]} \le t_{[2]} \le t_{[3]} \le \cdots \le t_{[n]}$$

Thus,

$$t_{[1]} = \min(\{t_i\}) \qquad \text{for} \qquad j = 1$$

and

$$t_{[j]} = \min\Big(\{t_i\} - \big\{t_{[1]}, \ldots, t_{[j-1]}\big\}\Big) \qquad \text{for} \qquad j = 2, 3, \ldots, n$$

where

$$t_{[n]} = \min\Big(\{t_i\} - \big\{t_{[1]}, \ldots, t_{[n-1]}\big\}\Big) = \max(\{t_i\}) \qquad \text{for} \qquad j = n$$

given $T_{k,n}$ is the life of a k-out-of-n system, where $k = 1, 2, 3, \ldots, n$. Because

$$T_{n,n} = t_{[1]}, \qquad T_{n-1,n} = t_{[2]}, \qquad T_{n-2,n} = t_{[3]}, \ldots$$
$$T_{2,n} = t_{[n-1]}, \qquad T_{1,n} = t_{[n]}$$

or

$$T_{k,n} = t_{[n-(k-1)]}$$

In case of $n = 4$, the life of a 1-out-of-4 system is $t_{[4]}$. The life of a 2-out-of-4 system is $t_{[3]}$. The life of a 3-out-of-4 system is $t_{[2]}$, and the life of a 4-out-of-4 system is $t_{[1]}$. Details are available in Ref. 9. The extensive literature review in Ref. 9 has indicated that the k-out-of-n structure has not been studied in a manner presented in this Note.

An example is now presented. For $n = 4$, let the random lives of the units be $t_1 = 1000.33$, $t_2 = 545.73$, $t_3 = 1620.04$, and $t_4 = 1180.01$ time units. Then

$$t_{[1]} = \min(\{t_1, t_2, t_3, t_4\}) = t_2 = 545.73$$

$$t_{[2]} = \min\Big(\{t_1, t_2, t_3, t_4\} - \big\{t_{[1]}\big\}\Big) = \min(\{t_1, t_2, t_3, t_4\} - \{t_2\})$$

$$t_{[2]} = \min(\{t_1, t_3, t_4\}) = t_1 = 1000.33$$

$$t_{[3]} = \min\Big(\{t_1, t_2, t_3, t_4\} - \big\{t_{[1]}, t_{[2]}\big\}\Big)$$

$$t_{[3]} = \min(\{t_1, t_2, t_3, t_4\} - \{t_2, t_1\})$$

$$t_{[3]} = \min(\{t_3, t_4\}) = t_4 = 1180.01$$

$$t_{[4]} = \max(\{t_1, t_2, t_3, t_4\}) = t_3 = 1620.04$$

$$T_{1,4} = t_{[4-(1-1)]} = t_{[4]} = t_3 = 1620.04$$

$$T_{2,4} = t_{[4-(2-1)]} = t_{[3]} = t_4 = 1180.01$$

$$T_{3,4} = t_{[4-(3-1)]} = t_{[2]} = t_1 = 1000.33$$

$$T_{4,4} = t_{[4-(4-1)]} = t_{[1]} = t_2 = 545.73$$

This example illustrates the basis for the simulation process. Random lives are drawn from respective distributions and used as just shown. Then, statistics are collected and analyzed.

Analytical Results

The preceding example may be valid just for one of the many identical spacecraft that will be traveling in the future. To make policy decisions, the analyst needs probability models that describe the behavior of all such spacecraft. Analytical results are always preferable to simulation because analytical formulas are applicable to any input data. Simulation has a distinct advantage of always providing the decision maker with a PDF or the histogram of the output of interest. This is often not possible when analytical work is complicated. This section provides expressions developed for the following parameters of the output or the distribution of interest: expected value, standard deviation, and median. In addition, the reliability and PDF formulas are derived. Standard probability theorems are used.

Distribution of the Smallest Values (4-out-of-4 System Case)

Exponential Unit Lives

The PDF and CDF for each engine are

$$f(t) = \lambda e^{-\lambda t} \qquad \text{and} \qquad F(t) = 1 - e^{-\lambda t} \qquad \lambda, t \ge 0$$

where λ is the failure rate or the inverse of the mean life θ. Functions R, g, and h refer to respective reliability and PDF expressions:

$$R_{\min}(t) = [1 - (1 - e^{-\lambda t})]^4 = e^{-4\lambda t} \tag{1}$$

$$g_{\min}(t) = 4\lambda e^{-\lambda t}[1 - (1 - e^{-\lambda t})]^{4-1} = 4\lambda e^{-4\lambda t} \tag{2}$$

$$E_{\min}(t) = \tfrac{1}{4}\lambda \tag{3}$$

$$\sigma^2_{\min_T} = \int_0^\infty 4\lambda t^2 e^{-4\lambda t}\,\mathrm{d}t - \left(\frac{1}{4\lambda}\right)^2 = \frac{1}{16\lambda^2} \tag{4}$$

$$t_{\min_{\text{median}}} = F^{-1}\Big[1 - (0.50)^{\frac{1}{4}}\Big] = 0.173287/\lambda = 0.173287\theta \tag{5}$$

Uniform Unit Lives

$$f(t) = \begin{cases} [1/(b-a)] & \text{where} \qquad a \le t \le b \\ 0 & \text{elsewhere} \end{cases}$$

and

$$F(t) = \begin{cases} 0, & t < a \\ (t-a)/(b-a), & a \le t < b \\ 1, & t \ge b \end{cases}$$

$$R_{\min}(t) = \{1 - [(t-a)/(b-a)]\}^4 = [(b-t)/(b-a)]^4 \tag{6}$$

$$g_{\min}(t) = \frac{4(b-t)^3}{(b-a)^4} \tag{7}$$

$$E_{\min}[t] = (4a+b)/5 \tag{8}$$

$$\sigma^2_{\min_T} = E_{\min}[t^2] - \left[\frac{4a+b}{5}\right]^2 = \frac{2(b-a)^2}{75} \tag{9}$$

$$t_{\min_{\text{median}}} = b - (0.50)^{\frac{1}{4}}(b-a) \tag{10}$$

Normal Unit Lives

$$f(t) = (1/\sqrt{2\Pi}\sigma)\exp\Big\{-\tfrac{1}{2}[(t-\mu)/\sigma]^2\Big\}$$

Table 1 Extreme value distribution parameters

Component life distribution	Parameter	Minimum extreme (4-out-of-4 case)		Maximum extreme (1-out-of-4 case)	
		Simulation	Analytical	Simulation	Analytical
Exponential	Mean	751.35	750	6251.26	6250
	Std. dev.	751.62	750	3597.59	3579.46
	Median	521.44	519.86	5505.53	5514.60
Uniform	Mean	2700.29	2700	3299.81	3300
	Std. dev.	163.32	163.30	163.56	163.30
	Median	2659.55	2659.10	3340.41	3340.90
Normal	Mean	2481.45	2485.32	3510.53	3514.69
	Std. dev.	350.83	350.61	354.04	350.62
	Median	2500.38	2500.93	3492.02	3499.07

and

$$F(t) = \Phi[(t-\mu)/\sigma]$$

$$R_{\min}(t) = \{1 - \Phi[(t-\mu)/\sigma]\}^4 \tag{11}$$

$$g_{\min}(t) = 4\Phi[(t-\mu)/\sigma]\{1 - \Phi[(t-\mu)/\sigma]\}^3 \tag{12}$$

$$E_{\min}[t] = \int_0^\infty 4t\,\Phi\left(\frac{t-\mu}{\sigma}\right)\left[1 - \Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t \tag{13}$$

$$\sigma^2_{\min_T} = \int_0^\infty 4t^2\Phi\left(\frac{t-\mu}{\sigma}\right)\left[1 - \Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t - \left\{\int_0^\infty 4t\,\Phi\left(\frac{t-\mu}{\sigma}\right)\left[1 - \Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t\right\}^2 \tag{14}$$

$$t_{\min_{\text{median}}} = \mu + (1.41421)\,\sigma\,\text{erf}^{-1}(-0.681793) \tag{15}$$

where

$$\text{erf}^{-1}(-0.681793) = -0.705798$$

Distribution of the Largest Values (1-out-of-4 System Case)

Exponential Unit Lives

$$R_{\max}(t) = 1 - [1 - e^{-\lambda t}]^4 \tag{16}$$

$$h_{\max}(t) = 4\lambda e^{-\lambda t}[1 - e^{-\lambda t}]^3 \tag{17}$$

$$E_{\max}[t] = 25/12\lambda \tag{18}$$

$$\sigma^2_{\max_T} = \frac{205}{144\lambda^2} \tag{19}$$

$$t_{\max_{\text{median}}} = 1.8382\bar{\theta} \tag{20}$$

Uniform Unit Lives

$$R_{\max}(t) = 1 - [(t-a)/(b-a)]^4 \tag{21}$$

$$h_{\max}(t) = \frac{4(t-a)^3}{(b-a)^4} \tag{22}$$

$$E_{\max}[t] = (a+4b)/5 \tag{23}$$

$$\sigma_{\max_T} = [(b-a)/5]\sqrt{\tfrac{2}{3}} \tag{24}$$

$$t_{\max_{\text{median}}} = a + (0.50)^{\frac{1}{4}}(b-a) \tag{25}$$

Normal Unit Lives

$$R_{\max}(t) = 1 - \{\Phi[(t-\mu)/\sigma]\}^4 \tag{26}$$

$$h_{\max}(t) = 4\Phi[(t-\mu)/\sigma]\{\Phi[(t-\mu)/\sigma]\}^3 \tag{27}$$

$$E_{\max}[t] = \int_0^\infty 4t\,\Phi\left(\frac{t-\mu}{\sigma}\right)\left[\Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t \tag{28}$$

$$\sigma^2_{\max_T} = \int_0^\infty 4t^2\Phi\left(\frac{t-\mu}{\sigma}\right)\left[\Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t - \left\{\int_0^\infty 4t\,\Phi\left(\frac{t-\mu}{\sigma}\right)\left[\Phi\left(\frac{t-\mu}{\sigma}\right)\right]^3 \mathrm{d}t\right\}^2 \tag{29}$$

$$t_{\max_{\text{median}}} = \mu + (1.41421)\sigma\,\text{erf}^{-1}(0.681793) \tag{30}$$

where

$$\text{erf}^{-1}(0.681793) = 0.705798$$

Simulation Experiments

Simulation was performed to confirm the analytical parameter expressions as well as to illustrate that simulation is an excellent alternative to analytical work. Table 1 shows the results where 26,000 (default) iterations were used. Note that the given parameters show the mean, standard deviation, and median of 26,000 extreme observations. Each such run was repeated using nine different random seeds, but there was no meaningful difference in the final values reported in Table 1. In both uniform and normal component life distribution cases, the resulting mean and median values are very close both for simulation and analytical results. Normal distribution may then be suggested as a good and quick estimate to describe the resulting PDF, but this would be incorrect in case of uniform component lives. If Eqs. (7) and (22) are plotted, the resulting PDF curves are highly skewed in spite of near equal mean and median values. Simulation results confirm this. As shown in Table 1, standard deviations of minimum and maximum extreme value distributions are equal when the underlying components are uniform and normal. Equation pairs (9) and (24) and (14) and (29) are also identical.

Each simulated output was submitted to chi-square goodness-of-fit test to identify the best PDF to represent the output. Table 2 shows the best fitting distributions after all necessary statistical confirmations were performed. Simulation may not be necessary in some cases because analytical work could be very straightforward when the underlying random variables are independent. Analytical work may quickly become complicated when one or more realistic situations must be considered. For example, random variables may be dependent and/or correlated if the performance of one engine affects the others. The spacecraft may have standby engines for use in the event of a malfunction. Some engines may be used at lesser capacity until some failure occurs with primary engines. These are some of the cases when simulation may have to be used regardless of the available analytical results. Even with a simple analytical case as in this study, simulation provides one more additional set of information (the minimum and maximum values or the extremes of the extreme distributions) for which no analytical formula can be developed.

Applications

Simulation application gives an additional information not available from analytical approach. As shown in Table 2, the analyst can

Table 2 Distribution identification for extreme value distributions

Component life	Minimum extreme (4-out-of-4 case)	Maximum extreme (1-out-of-4 case)
Exponential	Gamma (E)[a]	Gamma (E)
Location parameter	0.02	375.86
Scale parameter	731.36	2135.82
Shape parameter	1.01	2.72
Uniform	Weibull (E)	Extreme value type A
Location parameter	2500.03	3370.41
Scale parameter	206.28	120.79
Shape parameter	1.13	——
Normal	Weibull (E)	Log normal
Location parameter	982.34	0.0
Scale parameter	1631.93	8.16
Shape parameter	4.77	0.10

[a]E is extended version of the distribution specified.

obtain the PDF of the final random variable of interest. For example, if each engine has a normally distributed life, the maximum life in this scenario has a PDF that fits to a log-normal distribution with location, scale, and shape parameter values of 0, 8.16, and 0.10. Let us assume that the trip requires 3800 time units. What is the probability that the mission will be accomplished as one or more engines fail as time goes on? The reliability equation (26) is used to find $R(3800) = 0.2018$. For 2500 time units, the same reliability is 0.9996. Simulation runs[9] have ranged from a minimum of 2131 to a maximum of 5108 time units of duration.

These probabilities are exact answers. If the voyage requires 3800 time units of flight, then there is only 0.2018 probability of reaching the destination even though only one of the four engines is sufficient to power the spacecraft. Decision makers can use this information in determining whether and how to enhance the performance of the engines. If the analytical formula is not available, simulation results can be used. Log-normal CDF formula value or the probability of failure by time 3800, using the parameters shown in Table 2 for this case, is 0.794. Then, reliability is 0.2060 and is higher than the exact value of 0.2018. The very small difference between these two values is similar to the closeness between the simulated and analytical parameters shown in Table 1. If plotted, this case looks near normal as the shape parameter of log-normal is close to zero (0.10) suggesting this normality. In Table 2, the mean and median are also close. It is possible to use normal distribution with a parameter set of 3510.53 and 354.04 (simulation results) without having to identify the distribution itself. Then, $R(3800)$ is 0.2080 and is slightly more inaccurate than the one found using log-normal CDF.

For the 4-out-of-4 case, the reliability for 3800 time units or the probability that the flight can last at least 3800 time units is zero. There is no statistical chance for this mission to succeed although simulation results have ranged from 738 to 3807 time units.[9] In 1 (possibly 2 or 3) of the 26,000 replications the flight duration exceeded 3,800 time units. Statistically, the observation of 3807 and the possible few more between 3800 and 3807 have no significance. It is still true that the mission may have 1 or 2 chances out of 26,000 to succeed. The reliability drops to 0.5524 for 2500 time units compared to 0.9996 for the 1-out-of-4 case as would be expected if all four engines are needed. Table 2 shows that the simulated PDF of this 4-out-of-4 case is distributed according to a Weibull distribution with a parameter set of 982.34, 1631, and 4.77. The Weibull CDF for 2500 time units is 0.5070. Then, the reliability is 0.4930 and is considerably off from the exact value of 0.5524. If Weibull CDF is not used and a normal approximation is made, the reliability value is 0.479 and is more inaccurate.

Conclusion

To maintain the continuing pace of pioneering planetary missions, spacecraft must be highly reliable. This study has provided some simple probability and simulation-based results and concepts that can be used in this effort. Analytical expressions presented are simple, but none has been found readily available in relevant literature following an intense search. Rare texts have complicated mathematical developments, which are often intractable, and it is hard to find the needed information.

The closeness of the simulated and analytical parameters in Table 1 is encouraging for the value of simulation, which is simply an experiment. In one example given earlier, simulation did not perform well. That discrepancy can be partly attributed to inherent statistical errors in any goodness-of-fit tests as well as the simulation process itself.

Acknowledgments

This work is supported by the Colorado Space Grant Consortium after the initial work of Captain P. Siriphala.

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Maximum-Payload Transfers to Geosynchronous Orbit Using Arcjet Thrusters

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Introduction

THE payload benefits associated with the use of electric propulsion (EP) for performing near-Earth-orbit transfers has been investigated by several authors.[1–3] However, the utilization of a low-thrust engine for transferring a payload from low Earth orbit (LEO) to geosynchronous orbit (GEO) is a far-term application given the current level of technology of electric propulsion.[4] Current operational uses of EP include on-orbit maneuvers such as stationkeeping and drag make-up.[4] A potential current or near-term application of electric propulsion involves GEO orbit circularization in the case of the chemical apogee engine failure. Geosynchronous spacecraft are usually injected into an elliptical geosynchronous transfer orbit (GTO) with an apogee at GEO altitude and a perigee at LEO altitude. In 1989, the GSTAR-3 satellite utilized the hydrazine resistojet engine designed for on-orbit stationkeeping to circularize the GTO after the apogee engine failed.[4]

In this Note, the use of a combined chemical–electric propulsion system for a LEO–GEO transfer is investigated. The proposed mission scenario involves a chemical insertion into GTO, followed

Received Oct. 2, 1996; revision received March 3, 1997; accepted for publication March 7, 1997.

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